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On observation of time-delay systems with unknown inputs

G. Zheng , J-P. Barbot , D. Boutat, T. Floquet, J.-P. Richard

***Abstract*—Causal and non-causal observability are discussed in this paper for nonlinear time-delay systems with unknown inputs. Using the theory of non-commutative rings and the algebraic framework introduced by *Xia et al*, the nonlinear time-delay system is transformed into a suitable canonical form to solve the problem. A necessary and sufficient condition is given to guarantee the existence of a change of coordinates leading to such a form.**

***Index Terms*—Time-delay systems, Observability, Causality, Canonical form**

I. INTRODUCTION

OBSERVATION or estimation is important issue in control theory. For nonlinear systems without delays, the observability problem has been exhaustively studied, and has been characterized in [16], [21], [32] from a differential point of view, and in [8] from an algebraic point of view. For observable systems, many types of nonlinear observers have been proposed, such as high-gain observers in [13], algebraic observers in [3], [17], sliding mode observers in [12], [35] and the references therein.

However, unlike nonlinear systems without delays, the analysis of properties for time-delay system is more complicated (see the surveys [29] and [30]). For linear time-delay systems, various aspects of the observability problem have been studied in the literature, using different methods such as the functional analytic approach [4] or the algebraic approach [5], [11], [33]. The theory of non-commutative rings has been applied to analyze nonlinear time-delay systems firstly in [25] for the disturbance decoupling problem of nonlinear time-delay system, for observability of nonlinear time-delay systems with known inputs in [34], for identifiability of parameter for nonlinear time-delay systems in [36], and for state elimination and delay identification of nonlinear time-delay systems in [1]. Concerning the observer design for linear and nonlinear time-delay systems, the interested reader can refer to [7], [15], [18], [22], [28] and the references therein.

Most of those works are focused on time-delay systems with known inputs. However, in practical case, the input may be unknown. Thus one needs to study the state observability, and under which conditions the unknown input can be estimated. The first effort was done in [22] to extend Singh's inversion algorithm to nonlinear time-delay systems. Based on the algebraic framework proposed in [34], this paper deals with the causal and non-causal estimation analysis of the states and unknown inputs of nonlinear time-delay system. The issue of nonlinear observer design for the studied system is not treated in this paper.

This paper is organized as follows: In section II, we recall the algebraic framework introduced in [34], give some definitions and generalize the notation of the Lie derivative for time-delay systems with unknown inputs. Section III presents an observability canonical form for a general class of nonlinear time-delay systems, for which the causal and non-causal observabilities are discussed. In the same section, an illustrative example is given in order to highlight the proposed results.

II. ALGEBRAIC FRAMEWORK, NOTATIONS AND DEFINITIONS

In this paper, it is assumed that the delays are commensurable, that is all the delays are multiples of an elementary delay τ. Under this assumption, the considered nonlinear time-delay system is described as follows:

$$\begin{cases} \dot{x} = f(x(t-i\tau), i \in S_-) \\ \quad + \sum_{j=0}^{s} g^j(x(t-i\tau), i \in S_-)u(t-j\tau) \\ y = h(x(t-i\tau), i \in S_-) \\ x(t) = \psi(t), u(t) = \varphi(t), t \in [-s\tau, 0] \end{cases} \tag{1}$$

where $x \in W \subset R^n$ denotes the state variables, $u = [u_1, \cdots, u_m]^T \in R^m$ is the unknown input, $y \in R^p$ is the measurable output, with $p \geq m$. The integer i belongs to the finite set $S_- = \{0, 1, \cdots, s\}$. f, g^j and h are meromorphic functions[1] of $\{x(t), \cdots, x(t-s\tau)\}$. $\psi \in \mathcal{C}([-s\tau, 0], R^n)$ and $\varphi \in \mathcal{C}([-s\tau, 0], R^m)$ are the initial functions for x and u where $\mathcal{C}([-s\tau, 0], R^j)$ is the Banach space of function mapping $[-s\tau, 0]$ into R^j. In this work, it is assumed that (1) is locally observable when $u = 0$ (see Definition 1 hereafter), and admits a unique solution[2] and sufficiently differentiable outputs.

Based on the algebraic framework introduced in [34], let $\mathcal{K}$ be the field of meromorphic functions of a finite number of the variables from $\{x_j(t-i\tau), j \in [1, n], i \in S_-\}$. With the standard differential operator[3] d, define the vector space $\mathcal{E}$ over $\mathcal{K}$:

$$\mathcal{E} = span_{\mathcal{K}}\{d\xi : \xi \in \mathcal{K}\}$$

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[1] i.e. quotients of convergent power series with real coefficients [6], [34].

[2] Note the right hand-side of system (1) as $\bar{f}(x_\tau)$, if it is continuous with respect to its arguments, then there exists a solution for (1). Moreover, if it is locally Lipschitz, the solution is unique [9].

[3] The standard differential operator d maps elements from $\mathcal{K}$ to $\mathcal{E}$, which is the vector space spanned by the $\{dx_j(t-i\tau), j \in [1, n], i \in S_-\}$ over $\mathcal{K}$.

Introduce the backward time-shift operator δ defined by

$$\delta^i\xi(t) = \xi(t-i\tau), \xi(t) \in \mathcal{K}, \text{ for } i \in Z^+ \tag{2}$$

and

$$\delta^i\left(a(t)d\xi(t)\right) = \delta^i a(t)\delta^i d\xi(t) = a(t-i\tau)d\xi(t-i\tau) \tag{3}$$

for $a(t)d\xi(t) \in \mathcal{E}$, and $i \in Z^+$.

Let $\mathcal{K}(\delta]$ denote the set of polynomials of the form

$$a(\delta] = a_0(t) + a_1(t)\delta + \cdots + a_{r_a}(t)\delta^{r_a} \tag{4}$$

where $a_i(t) \in \mathcal{K}$. The addition in $\mathcal{K}(\delta]$ is defined as usual, but the multiplication is given as

$$a(\delta]b(\delta] = \sum_{k=0}^{r_a+r_b}\sum_{i+j=k}^{i\le r_a, j\le r_b} a_i(t)b_j(t-i\tau)\delta^k \tag{5}$$

Note that $\mathcal{K}(\delta]$ satisfies the associative law and is a non-commutative ring (see [34]). However, it is proved that the ring $\mathcal{K}(\delta]$ is a left Ore ring[4] [20], [34], which enables to define the rank of a module over this ring. Let $\mathcal{M}$ denote the left-module over $\mathcal{K}(\delta]$: $\mathcal{M} = span_{\mathcal{K}(\delta]}\{d\xi, \xi \in \mathcal{K}\}$, where $\mathcal{K}(\delta]$ acts on $d\xi$ according to (2) and (3).

With the definition of $\mathcal{K}(\delta]$, the system (1) can be rewritten in a more compact form as follows:

$$\begin{cases} \dot{x} = f(x,\delta) + \sum_{i=1}^m G_i(x,\delta)u_i(t) \\ y = h(x,\delta) \\ x(t) = \psi(t), u(t) = \varphi(t), t \in [-s\tau, 0] \end{cases} \tag{6}$$

where $f(x,\delta) = f(x(t-i\tau), i \in S_-)$ and $h(x,\delta) = h(x(t-i\tau), i \in S_-)$ with entries belonging to $\mathcal{K}$, $G_i(x,\delta) = \sum_{j=0}^s g_i^j(x,\delta)\delta^j$ with entries belonging to $\mathcal{K}(\delta]$. It is assumed that $rank_{\mathcal{K}(\delta]}\frac{\partial h}{\partial x} = p$, which implies that $[h_1, \cdots, h_p]^T$ are independent functions of x and its backward shifts.

Similarly to observability definitions for nonlinear systems without delays given in [16] and in [8], a definition of observability for time-delay systems is given in [24]. A more generic definition is stated here as follows:

Definition 1: System (1) is locally observable if the state $x(t)$ can be expressed as a function of the output and its time derivatives with their backward and forward shifts. A locally observable system is locally causally observable if its state can be written as a function of the output and its derivatives with their backward shifts only. Otherwise, it is locally non-causally observable (and it depends also on the forward shifts).

In the same way, the following definition for the unknown inputs is given.

Definition 2: The unknown input $u(t)$ can be locally estimated if it can be written as a function of the output and its time derivatives with backward and forward shifts. The input can be locally causally estimated if u can be expressed as a function of the output and its time derivatives with backward shifts only. Otherwise, it can be non causally estimated (and it depends also on the forward shifts).

[4] A ring $\mathcal{K}(\delta]$ is called a left Ore ring, if for all $a(\delta] \in \mathcal{K}(\delta]$ and $b(\delta] \in \mathcal{K}(\delta]$, there exist $a'(\delta] \in \mathcal{K}(\delta]$ and $b'(\delta] \in \mathcal{K}(\delta]$ not both zero, such that $a'(\delta]a(\delta] = b'(\delta]b(\delta]$.

Remarks 1: $i)$ Definition 1 does not take into account the input and its successive derivatives, because the input is assumed to be unknown and just required to be continuous. Thus, this paper states that the estimation of unknown input is causal if it depends only on past and present information of the output.

$ii)$ For simplicity, this paper uses the notion of observability to refer to both observations of the states and unknown inputs, if there is no ambiguity. If the state and the unknown input are observable in the sense of Definition 1 and 2, they can be directly estimated through robust differentiators, such as those proposed in [3], [10], [17].

Definition 3: (Unimodular matrix) [24] The matrix $A \in \mathcal{K}^{n\times n}(\delta]$ is said to be unimodular over $\mathcal{K}(\delta]$ if it has a *left* inverse $A^{-1} \in \mathcal{K}^{n\times n}(\delta]$, such that $A^{-1}A = I_{n\times n}$.

Definition 4: (Change of coordinates) [24] $z = \phi(\delta, x) \in \mathcal{K}^{n\times 1}$ is a causal change of coordinates over $\mathcal{K}$ for the system (1) if there locally exist a function $\phi^{-1} \in \mathcal{K}^{n\times 1}$ and some constants $c_1, \cdots, c_n \in N$ such that $diag\{\delta^{c_i}\}x = \phi^{-1}(\delta, z)$. The change of coordinates is bicausal over $\mathcal{K}$ if $max\{c_i\} = 0$, that is $x = \phi^{-1}(\delta, z)$.

Note that the relative degree for nonlinear systems without delays is well defined via the Lie derivative (see [19]). Then many efforts have been done to extend the classical Lie derivative for nonlinear time-delay systems. In [14], [15], the authors defined the so-called delay relative degree for a class of nonlinear time-delay systems with only a single delay, by augmenting the dimension of the studied system. In [27], [26], the authors introduced the delayed state derivative and delayed state bracket, which are extensions of the conventional Lie derivative. However, those definitions are still built on the theory of commutative rings, which make the analysis of observability for nonlinear time-delay systems still complicated. Hence in what follows, we first characterize the relative degree and observability indices for nonlinear time-delay systems by extending the Lie derivative in the algebraic framework of [34] from a non-commutative ring point of view. Then, it is shown that some known results for systems without delays, such as canonical form, can be extended to systems with delays.

Let $f(x(t-j\tau))$ and $h(x(t-j\tau))$ for $0 \le j \le s$ be n and p dimensional vectors, respectively, with entries $f_r \in \mathcal{K}$ for $1 \le r \le n$ and $h_i \in \mathcal{K}$ for $1 \le i \le p$. Let

$$\frac{\partial h_i}{\partial x} = \left[\frac{\partial h_i}{\partial x_1}, \cdots, \frac{\partial h_i}{\partial x_n}\right] \in \mathcal{K}^{1\times n}(\delta] \tag{7}$$

where for $1 \le r \le n$, $\frac{\partial h_i}{\partial x_r} = \sum_{j=0}^s \frac{\partial h_i}{\partial x_r(t-j\tau)}\delta^j \in \mathcal{K}(\delta]$. Then the Lie derivative for nonlinear systems without delays can be extended to nonlinear time-delay systems in the framework of [34] as follows

$$L_f h_i = \frac{\partial h_i}{\partial x}(f) = \sum_{r=1}^n\sum_{j=0}^s \frac{\partial h_i}{\partial x_r(t-j\tau)}\delta^j(f_r) \tag{8}$$

with $h_i \in \mathcal{K}$. One can also define $L_{G_i}h_i = \frac{\partial h_i}{\partial x}(G_i)$.

Using the above definition of Lie derivative, the relative degree can then be defined.

Definition 5: (Relative degree) System (6) has relative degree $(\nu_1, \cdots, \nu_p)$ in an open set $W \subseteq R^n$ if, for $1 \le i \le p$, the following conditions are satisfied :

1) for all $x \in W$, $L_{G_j} L_f^r h_i = 0$, for all $1 \leq j \leq m$ and $0 \leq r \leq \nu_i - 2$;
2) there exists $x \in W$ such that $\exists j \in [1, m]$, $L_{G_j} L_f^{\nu_i - 1} h_i \neq 0$.

If for $1 \leq i \leq p$, 1) is satisfied for all $r \geq 0$, we set $\nu_i = \infty$.

Since (6) is locally observable when $u = 0$, one can define the so-called observability indices introduced in [21]. Let $\mathcal{F}_k := span_{\mathcal{K}(\delta]} \left\{ dh, dL_f h, \cdots, dL_f^{k-1} h \right\}$ for $1 \leq k \leq n$. It was shown that the filtration of $\mathcal{K}(\delta]$-module satisfies $\mathcal{F}_1 \subset \mathcal{F}_2 \subset \cdots \subset \mathcal{F}_n$. Define $d_1 = rank_{\mathcal{K}(\delta]} \mathcal{F}_1$, $d_k = rank_{\mathcal{K}(\delta]} \mathcal{F}_k - rank_{\mathcal{K}(\delta]} \mathcal{F}_{k-1}$ for $2 \leq k \leq n$ and let $k_i = card \{ d_k \geq i, 1 \leq k \leq n \}$. Then $(k_1, \cdots, k_p)$ are the observability indices and $\sum_{i=1}^{p} k_i = n$ since it is assumed that (6) is observable with $u = 0$. Since $rank_{\mathcal{K}(\delta]} \frac{\partial h}{\partial x} = p$, the observability indices $(k_1, \cdots, k_p)$ for $(h_1, \cdots, h_p)$ are well defined.

III. Canonical form and observability

In order to facilitate the analysis of the general time-delay system (6), this section first proposes a canonical form of the general system (6), and then analyzes the causal and non-causal observability for the proposed canonical form.

A. Canonical form

After having defined the relative degree and observability indices via the extended Lie derivative for nonlinear time-delay systems in the framework of non-commutative rings, an observable canonical form is derived in this section.

Theorem 1: Consider the system (6) with outputs $(y_1, \cdots, y_p)$ and the corresponding $(\rho_1, \cdots, \rho_p)$ with $\rho_i = \min\{k_i, \nu_i\}$ where k_i and ν_i are the observability indices and the relative degree indices, respectively. There exists a change of coordinates $\phi(x, \delta) \in \mathcal{K}^{n \times 1}$, such that (6) can be transformed into the following form:

$$\dot{z}_{i,j} = z_{i,j+1} \tag{9}$$

$$\dot{z}_{i,\rho_i} = V_i(x, \delta) = L_f^{\rho_i} h_i(x, \delta) + \sum_{j=1}^{m} L_{G_j} L_f^{\rho_i - 1} h_i(x, \delta) u_j \tag{10}$$

$$y_i = C_i z_i = z_{i,1} \tag{11}$$

$$\dot{\xi} = \alpha(z, \xi, \delta) + \beta(z, \xi, \delta) u \tag{12}$$

where $z_i = (z_{i,1}, \cdots, z_{i,\rho_i})^T = \left(h_i, \cdots, L_f^{\rho_i - 1} h_i \right)^T \in \mathcal{K}^{\rho_i \times 1}$, $\alpha \in \mathcal{K}^{\mu \times 1}, \beta \in \mathcal{K}^{\mu \times 1}(\delta]$ with $\mu = n - \sum_{j=1}^{p} \rho_j$ and $C_i = (1, 0, \cdots, 0) \in R^{1 \times \rho_i}$. Moreover if $k_i < \nu_i$, one has $V_i(x, \delta) = L_f^{\rho_i} h_i = L_f^{k_i} h_i$. ■

Proof: According to the definition of the k_i, one has $rank_{\mathcal{K}(\delta]} \frac{\partial \left[h_i, \cdots, L_f^{k_i - 1} h_i \right]^T}{\partial x} = k_i$. Since $\rho_i = \min\{k_i, \nu_i\}$, one gets $rank_{\mathcal{K}(\delta]} \frac{\partial \left[h_i, L_f h_i, \cdots, L_f^{\rho_i - 1} h_i \right]^T}{\partial x} = \rho_i$, which implies that the components of $z_i = \left(h_i, \cdots, L_f^{\rho_i - 1} h_i \right)^T$ are linearly independent over $\mathcal{K}(\delta]$. Denote $z = \left(z_1^T, \cdots, z_p^T \right)^T \in \mathcal{K}^{n-\eta}$, with $\eta = \sum_{j=1}^{p} \rho_j$, and choose η variables $\xi \in \mathcal{K}^{\eta}$, such that $\left(z^T, \xi^T \right)^T = \phi(x, \delta) \in \mathcal{K}^n$ is a well-defined change of coordinates, which transforms (6) into (9-12).

Moreover, according to the definition of ν_i, for all $1 \leq j \leq m$ and $0 \leq r < \nu_i - 1$, if $k_i < \nu_i$, then $\rho_i = k_i$ and one has $L_{G_j} L_f^{k_i - 1} h_i(x, \delta) = L_{G_j} L_f^{\rho_i - 1} h_i(x, \delta) = 0$, which yields $V_i(x, \delta) = L_f^{\rho_i} h_i = L_f^{k_i} h_i$. ■

B. Causal observability

For the subsystem (9-11), one can find observers in the literature ([12], [35]) to estimate z_i and $V_i(x, \delta)$, for $1 \leq i \leq p$, in finite time due to the triangular structure. However the zero dynamic part (12) of the proposed canonical form cannot be estimated. In the following, a sufficient condition under which the system (6) can be transformed into the canonical form (9 -12) without the zero dynamic is given. Then, causal observability is analyzed.

The right-hand side of (10) can be rewritten in the following compact form:

$$H(x, \delta) = \Psi(x, \delta) + \Gamma(x, \delta) u \tag{13}$$

with $H(x, \delta) = (V_1(x, \delta), \cdots, V_p(x, \delta))^T$, $\Psi(x, \delta) = \left(L_f^{\rho_1} h_1, \cdots, L_f^{\rho_p} h_p \right)^T$ and

$$\Gamma(x, \delta) = \begin{pmatrix} L_{G_1} L_f^{\rho_1 - 1} h_1 & \cdots & L_{G_m} L_f^{\rho_1 - 1} h_1 \\ \vdots & \ddots & \vdots \\ L_{G_1} L_f^{\rho_p - 1} h_p & \cdots & L_{G_m} L_f^{\rho_p - 1} h_p \end{pmatrix} \tag{14}$$

where $H(x, \delta) \in \mathcal{K}^{p \times 1}$, $\Psi(x, \delta) \in \mathcal{K}^{p \times 1}$ and $\Gamma(x, \delta) \in \mathcal{K}^{p \times m}(\delta]$. Assume that $rank_{\mathcal{K}(\delta]} \Gamma = m$. Since $\Gamma \in \mathcal{K}^{p \times m}(\delta]$ with $m \leq p$, according to Lemma 4 in [23], there exists a matrix $\Xi \in \mathcal{K}^{p \times p}(\delta]$ such that $\Xi \Gamma = \left[\bar{\Gamma}^T, \mathbf{0} \right]^T$, where $\bar{\Gamma} \in \mathcal{K}^{m \times m}(\delta]$ has full rank m. Introduce the set:

$$\Phi = \{ dh_1, \cdots, dL_f^{\rho_1 - 1} h_1, \cdots, dh_p, \cdots, dL_f^{\rho_p - 1} h_p \} \tag{15}$$

Then, the following theorem can be stated.

Theorem 2: Consider the system (6) with outputs $(y_1, \cdots, y_p)$ and the corresponding $(\rho_1, \cdots, \rho_p)$ with $\rho_i = \min\{k_i, \nu_i\}$ where k_i and ν_i are the observability indices and the relative degree indices, respectively. If $rank_{\mathcal{K}(\delta]} \Phi = n$, where Φ is defined in (15), then there exists a change of coordinates $\phi(x, \delta)$ such that (6) can be transformed into (9-12) with dim $\xi = 0$.

Moreover, if the change of coordinates is locally bicausal over $\mathcal{K}$, then the state $x(t)$ of (6) is locally causally observable, and if $\bar{\Gamma} \in \mathcal{K}^{m \times m}(\delta]$ is also unimodular over $\mathcal{K}(\delta]$, then the unknown input $u(t)$ of (6) can be locally causally estimated as well.

Proof: According to Theorem 1, the system (6) can be transformed into (9-12) by using the change of coordinates $(z, \xi) = \phi(x, \delta)$. Hence, if $rank_{\mathcal{K}(\delta]} \Phi = n$, where Φ defined in (15), one has $\sum_{j=1}^{p} \rho_j = n$, which implies that (6) can be transformed into (9-12) with dim $\xi = 0$ and the change of coordinates is given by $z = \phi(x, \delta)$ where $z = (z_i^T, \cdots, z_p^T)^T$ and $z_i = (h_i, \cdots, L_f^{\rho_i - 1} h_i)^T$.

Moreover, if $\phi(x,\delta) \in \mathcal{K}^{n\times 1}$ is locally bicausal over $\mathcal{K}$, one can write x as a function of y_i, its derivative and backward shift, which implies state x is locally causally observable.

Concerning the reconstruction of the unknown inputs, rewrite (13) as follows

$$\Gamma u = H(x,\delta) - \Psi(x,\delta) = \Upsilon(x,\delta). \tag{16}$$

Since $rank_{\mathcal{K}(\delta]}\Phi = n$ and x is causally observable, then $\Upsilon(x,\delta)$ is a vector of known meromorphic functions belonging to $\mathcal{K}$.

If $\bar{\Gamma} \in \mathcal{K}^{m\times m}(\delta]$ is unimodular over $\mathcal{K}(\delta]$, then there exists a matrix $\bar{\Gamma}^{-1} \in \mathcal{K}^{m\times m}(\delta]$ such that $\begin{bmatrix} \bar{\Gamma}^{-1} & \mathbf{0} \end{bmatrix} \Xi\Gamma = I_{m\times m}$ and $u = \begin{bmatrix} \bar{\Gamma}^{-1} & \mathbf{0} \end{bmatrix} \Xi\Upsilon$. Since $\bar{\Gamma}^{-1} \in \mathcal{K}^{m\times m}(\delta]$, $\Xi \in \mathcal{K}^{p\times p}$ and $\Upsilon \in \mathcal{K}^{p\times 1}$, then u is also causally observable. ■

C. Extended case for causal observability

For the case where the condition $rank_{\mathcal{K}(\delta]}\Phi = n$ in Theorem 2 is failed, a constructive algorithm was proposed in [2] to solve this problem for nonlinear systems without delays. The result of this subsection can be seen as an extension of the work [2] to treat the observation problem for time-delay systems with unknown inputs. The objective is to generate additional variables from the available measurement and unaffected by the unknown input such that an extended canonical form similar to (9)-(10) can be obtained for the estimation of the remaining state ξ.

For this, consider Φ as defined in (15). If $rank_{\mathcal{K}(\delta]}\Phi = j$, one can select j linearly independent vectors over $\mathcal{K}(\delta]$ from Φ, denoted as $\Phi = \{dz_1, \cdots, dz_j\}$. Note $\pounds = span_{R[\delta]}\{z_1, \cdots, z_j\}$ where $R[\delta]$ is the commutative ring of polynomials of δ with coefficients belonging to the field R and let $\pounds(\delta]$ be the set of polynomials of δ with coefficients over $\pounds$. Define the module spanned by the elements of Φ over $\pounds(\delta]$ as follows

$$\Omega = span_{\pounds(\delta]}\{\xi, \xi \in \Phi\}. \tag{17}$$

Define also $\mathcal{G} = span_{R[\delta]}\{G_1, \cdots, G_m\}$ and its *left* annihilator $\mathcal{G}^{\perp} = span_{R[\delta]}\{\omega \in \Omega \mid \omega g = 0, \forall g \in \mathcal{G}\}$. Based on the above definitions, let us state the main result of the paper.

Theorem 3: Consider the system (6) with outputs $y = (y_1, \cdots, y_p)^T$ and the corresponding $(\rho_1, \cdots, \rho_p)$ with $\rho_i = \min\{k_i, \nu_i\}$ where k_i and ν_i are the observability indices and the relative degree indices, respectively. Suppose $rank_{\mathcal{K}(\delta]}\Phi < n$ where Φ is defined in (15). There exist l new independent outputs over $\mathcal{K}$ suitable to the causal estimation problem if and only if $rank_{\mathcal{K}}\mathcal{H} = l$ where

$$\mathcal{H} = span_{R[\delta]}\{\omega \in \mathcal{G}^{\perp} \cap \Omega \mid \omega f \notin \pounds\}. \tag{18}$$

Moreover, the l additional outputs, denoted $\bar{y}_i$, $1 \le i \le l$, are given by: $\bar{y}_i = \omega_i f \mod \pounds$ where $\omega_i \in \mathcal{H}$.

Proof:

Denote $Q_i = \left[q_1^i, \cdots, q_p^i\right]$ as $1\times p$ vector with $q_j^i \in \mathcal{K}(\delta]$ for $1 \le j \le p$. One has

$$Q_i\Gamma = \left(Q_i \begin{bmatrix} \frac{\partial L_f^{\rho_1 - 1} h_1}{\partial x} \\ \vdots \\ \frac{\partial L_f^{\rho_p - 1} h_p}{\partial x} \end{bmatrix} \right) [G_1, \cdots, G_m]$$

because of the associativity law over $\mathcal{K}(\delta]$. Then according to the definition (7), one gets

$$Q_i\Gamma = \omega_i [G_1, \cdots, G_m] = \omega_i G$$

where $\omega_i = \sum_{c=1}^{n}\sum_{j=1}^{p} q_j^i \frac{\partial L_f^{\rho_j - 1} h_j}{\partial x_c} dx_c$.

Moreover, one can check that

$$\omega_i f = \left(Q_i \begin{bmatrix} \frac{\partial L_f^{\rho_1 - 1} h_1}{\partial x} \\ \vdots \\ \frac{\partial L_f^{\rho_p - 1} h_p}{\partial x} \end{bmatrix} \right) f = Q_i \begin{bmatrix} L_f^{\rho_1} h_1 \\ \vdots \\ L_f^{\rho_p} h_p \end{bmatrix} = Q_i\Psi.$$

According to (13), one has

$$Q_i H = Q_i(\Psi + \Gamma u) = \omega_i f + \omega_i G u \tag{19}$$

where $H = (V_1, \cdots, V_p)^T$ is a vector which can be estimated in finite time but is affected by the unknown input. Thus, one can generate l additional information suitable to solve the estimation problem if and only if one can find l independent $\{Q_1, \cdots, Q_l\}$ vectors over $\mathcal{K}(\delta]$ such that for each $Q_i = \{q_1^i, \cdots, q_p^i\}$ satisfying $q_j^i \in \pounds(\delta]$, one has $Q_i\Gamma = 0$ and $Q_i H \notin \pounds$.

Thus, one has to prove that the following conditions are equivalent:

1) there exist l row vectors $Q_i = \left[q_1^i, \cdots, q_p^i\right]$, with $q_j^i \in \pounds(\delta]$, such that $rank_{\mathcal{K}(\delta]}\{Q_1, \cdots, Q_l\} = l$, $Q_i\Gamma = 0$ and $Q_i H \notin \pounds$.
2) $rank_{\mathcal{K}}\mathcal{H} = l$ with $\mathcal{H}$ defined in (18).

Necessity: Suppose item 1) is satisfied, then according to (19), one has $Q_i\Gamma = \omega_i G = 0 \Rightarrow \omega_i \in \mathcal{G}^{\perp}$ and $Q_i H = \omega_i f \notin \pounds$. Thus $rank_{\mathcal{K}(\delta]}\{Q_1, \cdots, Q_l\} = l$ implies $rank_{\mathcal{K}}\{\omega_1, \cdots, \omega_l\} = l$. Since $L_f^{\rho_i - 1} h_i \in \pounds$ and $q_i \in \pounds(\delta]$, then $\omega_i = \sum_{c=1}^{n}\sum_{j=1}^{p} q_j^i \frac{\partial L_f^{\rho_j - 1} h_j}{\partial x_c} dx_c \in \Omega$. Thus, $\omega_i \in \mathcal{H}$ defined in (18).

Sufficiency: Suppose $rank_{\mathcal{K}}\mathcal{H} = l$. Then one can find l independent 1-forms over $\mathcal{K}$: $\{\omega_1, \cdots, \omega_l\}$ with $\omega_i \in \mathcal{G}^{\perp} \cap \Omega$ which implies there exist l independent vectors over $\mathcal{K}(\delta]$: $\{Q_1, \cdots, Q_l\}$ with entries belonging to $\pounds(\delta]$ such that $rank_{\mathcal{K}(\delta]}\{Q_1, \cdots, Q_l\} = l$, since for each Q_i one has $Q_i\Gamma = \omega_i G = 0$ and $Q_i H = \omega_i f \notin \pounds$. The variable $\bar{y}_i = Q_i H = \omega_i f \mod \pounds$ can be used as an additional output since

1) H can be estimated in finite time;
2) Q_i has entries in $\pounds(\delta]$;
3) $\bar{y}_i$ do not belong to the current set $\pounds$ of measured variables.

■

Remark 1: Theorem 3 gives a constructive way to treat the case where $rank_{\mathcal{K}(\delta]}\Phi < n$. Once additional new outputs are deduced according to Theorem 3, it enables to define a new Φ. If $rank_{\mathcal{K}(\delta]}\Phi = n$, Theorem 2 can then be applied. Otherwise, if $rank_{\mathcal{K}(\delta]}\Phi < n$ and if Theorem 3 is still valid, then one can still deduce new outputs for the studied system. Thus a “Check-Extend” procedure is iterated until one obtains $rank_{\mathcal{K}(\delta]}\Phi = n$.

D. Non-causal observability

The previous results can be extended to the case of non-causal observations of the state and the unknown inputs which can be very useful in some applications. For instance, many proposed delay feedback control methods can be applied for stabilizing nonlinear time-delay systems [31]. Furthermore, other applications, such as cryptography based on chaotic system, do not require real-time estimation, hence non-causal observations can still play an important role in those applications.

In order to treat the non-causal case, let us introduce the forward time-shift operator ∇, which is similar to the backward time-shift operator δ defined in Section II:

$$\nabla f(t) = f(t+\tau)$$

and

$$\nabla^i \delta^j f(t) = \delta^j \nabla^i f(t) = f\left(t-(j-i)\tau\right)$$

for $i, j \in Z^+$.

Following the same principle of Section II, denote $\bar{\mathcal{K}}$ the field of meromorphic functions of a finite number of variables from $\{x_j(t-i\tau), j \in [1,n], i \in S\}$ where $S = \{-s, \cdots, 0, \cdots, s\}$ is a finite set of constant. One has $\mathcal{K} \subseteq \bar{\mathcal{K}}$. Denote $\bar{\mathcal{K}}(\delta, \nabla]$ the set of polynomials of the form:

$$\begin{array}{rcl} a(\delta,\nabla] & = & \bar{a}_{r_{\bar{a}}} \nabla^{r_{\bar{a}}} + \cdots + \bar{a}_1 \nabla \\ & & +a_0(t) + a_1(t)\delta + \cdots + a_{r_a}(t)\delta^{r_a} \end{array} \quad (20)$$

with $a_i(t)$ and $\bar{a}_i(t)$ belonging to $\bar{\mathcal{K}}$. Keep the usual definition of addition for $\bar{\mathcal{K}}(\delta, \nabla]$ and define the multiplication as follows:

$$\begin{array}{rcl} a(\delta,\nabla]b(\delta,\nabla] & = & \sum\limits_{i=0}^{r_a}\sum\limits_{j=0}^{r_b} a_i\delta^i b_j \delta^{i+j} + \sum\limits_{i=0}^{r_a}\sum\limits_{j=1}^{r_{\bar{b}}} a_i \delta^i \bar{b}_j \delta^i \nabla^j \\ & + & \sum\limits_{i=1}^{r_{\bar{a}}}\sum\limits_{j=0}^{r_b} \bar{a}_i \nabla^i b_j \nabla^i \delta^j + \sum\limits_{i=1}^{r_{\bar{a}}}\sum\limits_{j=1}^{r_{\bar{b}}} \bar{a}_i \nabla^i \bar{b}_j \nabla^{i+j} \end{array} \quad (21)$$

It is clear that $\mathcal{K}(\delta] \subseteq \bar{\mathcal{K}}(\delta, \nabla]$ and that the ring $\bar{\mathcal{K}}(\delta, \nabla]$ possesses the same properties as $\mathcal{K}(\delta]$. Thus a module $\bar{\mathcal{M}}$ can be also defined over $\bar{\mathcal{K}}(\delta, \nabla]$ as $\bar{\mathcal{M}} = span_{\bar{\mathcal{K}}(\delta,\nabla]}\{d\xi, \xi \in \bar{\mathcal{K}}\}$.

Given the above definitions, Theorem 2 can then be extended as follows in order to deal with non-causal observability for nonlinear time-delay systems.

Theorem 4: Consider the system (6) with outputs $(y_1, \cdots, y_p)$ and the corresponding $(\rho_1, \cdots, \rho_p)$ with $\rho_i = \min\{k_i, \nu_i\}$ where k_i and ν_i are the observability indices and the relative degree indices, respectively. If $rank_{\mathcal{K}(\delta]}\Phi = n$, where Φ is defined in (15), then there exists a change of coordinates $\phi(x,\delta)$ such that (6) can be transformed into (9-12) with dim $\xi = 0$.

Moreover, if the change of coordinates is locally bicausal over $\bar{\mathcal{K}}$, then the state $x(t)$ of (6) is at least locally non-causally observable, and if $\bar{\Gamma} \in \mathcal{K}^{m\times m}(\delta]$ is also unimodular over $\bar{\mathcal{K}}(\delta, \nabla]$, then the unknown input $u(t)$ of (6) can be at least locally non-causally estimated as well.

Proof: If the change of coordinates $z = \phi(x,\delta) \in \mathcal{K}^{n\times 1} \subseteq \bar{\mathcal{K}}^{n\times 1}$ is locally bicausal over $\bar{\mathcal{K}}$, then there exist $\phi^{-1} \in \bar{\mathcal{K}}^{n\times 1}$ and some constants $c_1, \cdots, c_n$ such that $\mathcal{T}x = \phi^{-1}(z,\delta)$ where $\mathcal{T} = diag\{\delta^{c_1}, \cdots, \delta^{c_n}\}$. Thus one can define the matrix $\mathcal{T}^{-1} = diag\{\nabla^{c_1}, \cdots, \nabla^{c_n}\} \in \bar{\mathcal{K}}^{n\times n}(\delta, \nabla]$, such that $x = \mathcal{T}^{-1}\phi^{-1}(\delta, z) \in \bar{\mathcal{K}}^{n\times 1}$, which means that x is at least locally non-causally observable.

For the estimation of $u(t)$, if $\bar{\Gamma}$ is unimodular over $\bar{\mathcal{K}}(\delta, \nabla]$, following the same procedure as in Theorem 2, one gets $u = \left[\begin{array}{cc} \bar{\Gamma}^{-1} & \mathbf{0} \end{array}\right] \Xi \Upsilon$. In this case, since $\bar{\Gamma}^{-1} \in \bar{\mathcal{K}}(\delta, \nabla]$, $\Xi \in \mathcal{K}^{p\times p}(\delta]$ and $\Upsilon \in \mathcal{K}^{p\times 1}$, u is at least non-causally observable. ■

E. Example

Here is given an illustrative example in order to highlight the proposed results in the case of causal observability.

Consider the following system

$$\left\{ \begin{array}{l} \dot{x}_1 = -\delta x_1^2 + \delta x_4 u_1, \dot{x}_2 = -x_1^2 \delta x_3 + x_4 \\ \dot{x}_3 = x_2 - x_1^2 \delta x_4 u_1, \dot{x}_4 = u_2 \\ y_1 = x_1, y_2 = x_1 \delta x_1 + x_3 \end{array} \right. \quad (22)$$

One can check that $\nu_1 = k_1 = 1$, $\nu_2 = 1$ and $k_2 = 3$, yielding $\rho_1 = \rho_2 = 1$ and $\Phi = \{dx_1, (\delta x_1 + x_1 \delta)dx_1 + dx_3\}$. One has $rank_{\mathcal{K}(\delta]}\Phi = 2 < n$.

Set $\mathcal{G} = span_{R[\delta]}\{G_1, \cdots, G_m\}$. Then one has $\mathcal{G}^\perp = span_{R[\delta]}\left\{x_1^2 dx_1 + dx_3, dx_2\right\}$. Since $rank_{\mathcal{K}(\delta]}\Phi = 2$, $\pounds = span_{R[\delta]}\left\{x_1, x_1\delta x_1 + x_3\right\}$ and $\Omega = span_{\pounds(\delta]}\left\{dx_1, dx_3\right\}$. One obtains

$$\Omega \cap \mathcal{G}^\perp = span_{\pounds(\delta]}\left\{x_1^2 dx_1 + dx_3\right\}.$$

From the definition of $\mathcal{H}$ in (18), one can check that $rank_{\mathcal{K}}\mathcal{H} = 1$, which gives the one-form $\omega = x_1^2 dx_1 + dx_3$, satisfying $\omega \in \Omega \cap \mathcal{G}^\perp$ and $\omega f = -x_1^2 \delta x_1^2 + x_2 \notin \pounds$. Thus, a new output $\bar{y}_1 = h_3$ is given by

$$\begin{array}{rl} \bar{y}_1 & = h_3 = \omega f \mod \pounds \\ & = x_2 = y_1^2 \delta y_1^2 + (y_1^2 - y_1\delta)\dot{y}_1 + \dot{y}_2 \end{array} \quad (23)$$

Although $\dot{y}_1$ and $\dot{y}_2$ contain u and cannot be derivable, however their combination in (23) makes $\bar{y}_1$ not depend on u, thus it is derivable and can be used for state reconstruction. For the new output $\bar{y}_1$, one has $\nu_3 = 2$ and $k_3 = 2$, thus $\rho_3 = 2$. Finally, one obtains the new Φ as follows:

$$\begin{array}{rl} \Phi = & \{dx_1, (\delta x_1 + x_1\delta)dx_1 + dx_3, dx_2, \\ & -2x_1\delta x_3 dx_1 - x_1^2 \delta dx_3 + dx_4\}. \end{array}$$

It can be checked that $rank_{\mathcal{K}(\delta]}\Phi = 4$, and the new $\pounds$ is

$$\pounds = span_{R[\delta]}\{x_1, x_1\delta x_1 + x_3, x_2, -x_1^2\delta x_3 + x_4\}.$$

This gives the following change of coordinates

$$z = \phi(x,\delta) = \left(x_1, x_1\delta x_1 + x_3, x_2, -x_1^2 \delta x_3 + x_4\right)^T.$$

It is easy to check that it is bicausal over $\mathcal{K}(\delta]$, since

$$x = \phi^{-1} = \left(z_1, z_3, z_2 - z_1\delta z_1, z_4 + z_1^2\delta z_2 - z_1^2 \delta z_1 \delta^2 z_1\right)^T.$$

When $t \geq 2\tau$, one gets the following estimations of states:

$$\left\{ \begin{array}{l} x_1 = y_1, x_2 = \bar{y}_1, x_3 = y_2 - \delta y_1 \\ x_4 = -y_1^3 \delta^2 y_1 + y_1^2 \delta y_2 + \dot{\bar{y}}_1 \end{array} \right.$$

with $\bar{y}_1$ defined in (23).

Moreover, the matrix Γ with the new output $\bar{y}_1$ is $\Gamma = \left(\begin{array}{cc} \delta x_4, & 0 \\ \delta x_1 \delta x_4 + x_1\delta^2 x_4 \delta - x_1^2 \delta x_4, & 0 \\ -2x_1\delta x_3 \delta x_4 + x_1^2 \delta x_1^2 \delta^2 x_4 \delta, & 1 \end{array} \right)$ with $rank_{\mathcal{K}(\delta]}\Gamma = 2$.

One can find matrices $\Xi = \begin{pmatrix} 1 & 0 & 0 \\ -\delta x_1 - x_1\delta + x_1^2 & 1 & 0 \\ 2x_1\delta x_3 - x_1^2\delta x_1^2\delta & 0 & 1 \end{pmatrix}$, $\bar{\Gamma} = \begin{pmatrix} \delta x_4 & 0 \\ 0 & 1 \end{pmatrix}$, and $\bar{\Gamma}^{-1} = \begin{pmatrix} \frac{1}{\delta x_4} & 0 \\ 0 & 1 \end{pmatrix}$ such that $\begin{bmatrix} \bar{\Gamma}^{-1} & \mathbf{0} \end{bmatrix} \Xi\Gamma = I_{2\times 2}$. Consequently, according to Theorem 2, u_1 and u_2 can be causally estimated. When $t \geq 3\tau$, a straightforward computation yields the following estimates for the unknown inputs:

$$\begin{cases} u_1 = \frac{\dot{y}_1 + \delta y_1^2}{-\delta y_1^3 \delta^3 y_1 + \delta y_1^2 \delta^2 y_2 + \delta \dot{y}_1} \\ u_2 = -3y_1^2\delta^2 y_1 \dot{y}_1 - y_1^3 \delta^2 \dot{y}_1 + 2y_1 \delta y_2 \dot{y}_1 + y_1^2 \delta \dot{y}_2 + \dddot{y}_1 \end{cases}$$

IV. Conclusion

In this paper, a generic definition of observability for time-delay systems with unknown inputs, covering causal and non-causal observability, has been introduced. The relative degree and observability indices for nonlinear time-delay systems have been defined based on the notation of the Lie derivation in the framework of non-commutative rings. Then, an observable canonical form for time-delay systems, as well as sufficient conditions to guarantee the causal and non-causal observations of states and unknown inputs of time-delay systems, have been given.

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